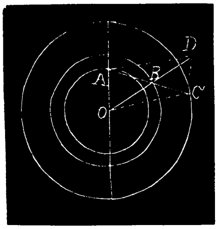
Through A and B draw AC intersecting outer circumference at C. Then $AB=BC$, and AC is the required line.



PROOF. $OB=BD$, $OC=AD$, and $\angle OBC=\angle ABD$. $\therefore \triangle OBC=\triangle ABD$. $\therefore AB=BC$. Also, $OADC$ is a parallelogram, of which the diagonals OD and AC bisect each other at B.

COROLLARY 1. Put a, b, and c=the respective radii of the three concentric circles, taking $a<b<c$, and put $2d$=line AC. Then, from the relation of the diagonals to the sides of a parallelogram,

$$2d=\sqrt{(2c^2+2a^2-4b^2)}.$$ GRUBER.

COROLLARY 2. The problem is possible only for $c-b=b-a$ to $c-b=b+a$, or for $2b=c+a$ to $2b=c-a$. Whence the limits of $2d$ are $c-a$ and $c+a$, the parallelogram in either case reducing to a straight line. GRUBER.

COROLLARY 3. The point D is without the outer circle for $2b>c$. When $2b=c$, D lies in the outer circumference. When $2b<c$, D lies within the outer circle. GRUBER.

COROLLARY 4. When $2b>\sqrt{(c^2+3a^2)}$, $2d$ lies wholly without the inner circle. When $2b=\sqrt{(c^2+3a^2}$, $2d$ is tangent to the inner circumference. When $2b<\sqrt{(c^2+3a^2)}$, $2d$ is a secant of the inner circle. GRUBER.

Also solved by *G. B. M. ZERR*, *J. W. YOUNG*. *GEORGE R. DEAN*, *B. F. YANNEY*, *B. F. SINE*, *WALTER H. DRANE*; and *WM. K. NORTON*.

CALCULUS.

88. Proposed by JOHN M. ARNOLD, Crompton, R. I.

When a watch is wound up, the mainspring is closely coiled around a cylindrical piece called the hub of the barrel-arbor. When entirely run down the spring forms an annulus against the inner circumference of the barrel. Show that if the width of the annulus is a little more than one-fourth of the radius of the barrel, the spring will run the watch the greatest number of hours at one winding, the diameter of the hub being one-third the inside diameter of the barrel.

I. Solution by the PROPOSER.

Let R=radius of the barrel, r=radius of the hub, t=thickness of spring, x=width of the annulus when run down, y=width of the annulus when wound up, u=number of turns required to wind the spring.

Then x/t=number of coils of the spring when run down, and y/t=number of coils when wound up.

Hence $u=y/t-x/t$......(1).

It is evident that the area on the bottom of the barrel covered by the spring will be the same in the wound or unwound condition.

Hence $R^2\pi-(R-x)^2\pi-(r+y)^2\pi-r^2\pi$.

Reducing $2Rx-x^2=2ry+y^2$..... (2).

From (1) and (2), $tu=\pm\sqrt{[2Rx-x^2+r^2]}-r-x$.

Differentiating and equating to zero,

$$\frac{du}{dx}=\pm\frac{R-x}{t\sqrt{[2Rx-x^2+r^2]}}-1/t=0.$$

Reducing, $2x^2-4Rx=r^2-R^2$.

Whence $x=R\pm\sqrt{\left(\frac{R^2+r^2}{2}\right)}$.

Making $r=\frac{1}{3}R$, and taking the minus sign, $x=R(1-\sqrt{\frac{5}{9}})$.

$x=.25464R$, or a little more than one-fourth of the radius.

II. Solution by HENRY HEATON, M. Sc., Atlantic, Ia.

To secure the greatest result, the area occupied by the spring when wound or unwound must be one-half that between the hub and the inner circumference of the barrel. This area is $\frac{4}{9}r^2\pi$, and the area occupied by the hub and spring when the latter is wound, is $\frac{5}{9}r^2\pi$. Hence the radius of the circumference lying within the annulus is $r'=\frac{1}{3}r\sqrt{5}=.745r$.

$\therefore r-r'=.254r$.

III. Solution by P. H. PHILBRICK, C. E., Lake Charles, La.

Let a=area of the cross-section of the barrel; then $(\frac{1}{3})^2a=\frac{1}{9}a$=the area of the cross-section of the hub; $\frac{8}{9}a$=area around the hub, and $\frac{4}{9}a$=one-half of that area; $\frac{5}{9}a$=area of cross-section of hub and spring.

Hence both hub and spring occupy $\sqrt{\frac{5}{9}}=.7454$ of the radius of the barrel, and the unwound spring occupies $1-.7454=.2546$ of that radius.

MECHANICS.

81. Proposed by JAMES S. STEVENS, Professor of Physics, The University of Maine, Orono, Me.

Two iron spheres whose weights are a and b, and a is greater than b, are suspended over a frictionless pulley so that they move in a liquid medium of density δ. Assume that the density of the iron is δ', what would be the spaces passed over (downward by a and upward by b) in the first four seconds, if the spheres start from rest?

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let m=the mass$=(a-b)/g$......(1).

v=the velocity at time t, Rv^2=resistance. The resistance is the sum of the resistances for both spheres.

Let A=the greatest cross-section of $a=\frac{\sqrt[3]{[36a^2\pi g\delta'}}{4g\delta'}$.

Let B=the greatest cross-section of $b=\frac{\sqrt[3]{[36b^2\pi g\delta'}}{4g\delta'}$.